Agents-Model for Finance

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In this paper short reviews and some numerical results are given for the recent research developments in stock, foreign exchange and commodity markets from the physicist's point of view where the new branch of physics "econophysics" has been established.[1] These new fields are attractive for physicists mainly because many detailed data are available and the problems seem suitable for statistical methods.

Empirical findings for stock and foreign exchange markets[2] are

(i) that the distributions of logarithmic price changes show power law tails with an exponent $\alpha \approx 3$, so called fat tails,

(ii) that the price changes are correlated only over rather short time scales of typically $\tau \approx 4$ minutes, and

(iii) that the amplitude of the price changes, volatility has long-range correlation which vanishes as a power law with an exponent of $\gamma \approx 1/3$.

Similar results have been reported recently for the commodity markets as well,[3]

(i) an exponent $\alpha \approx 3$,

(ii) short time scales of typically $\tau \approx 3$ days or more, and

(iii) an exponent $\gamma \approx 0.54$.

A new theory is required to explain these discoveries which go beyond the efficient market hypothesis[4), 5)] for the following reasons. First, these results disagree with beliefs that the price changes for stock and foreign exchange markets obey the geometrical Brownian motion, exhibiting the Gaussian tail for its distribution. It should decay exponentially fast for the Gaussian tail. Second, they indicate the universal nature where

1) See for example http://www.unifr.ch/econophysics/ in the Internet for the latest development.

2) R. N. Mantegna and H. E. Stanley, "An Introduction to Econophysics Correlations and Complexity in finance", Cambridge University Press, Cambridge, UK, 2000.

3) Kaushik Matia, Luis A. Nunes Amaral, Stephen P. Goodwin and H. Eugene Stanley, "Non-Lévy Distribution of Commodity Price Fluctuations"(2002), preprint cond-mat/0202028.

4) P. A. Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly", Industrial Management Rev. 6(1965)41.

5) E. F. Fama, "Efficient Capital Markets: II", J. Finance **46**(1991)1575.

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quite different markets obey the similar regularities. Finally, the value of the exponents $\alpha \approx 3$ excludes the application of Lévy stable distribution which was earlier proposed for commodity markets and others.

There have been many approaches to understand the price changes in finance. Among them there are empirical methods which practitioners of financial institutions use such as ARCH models,[6] Black-Scholes formula,[7, 8] etc., some of them as early as the efficient market hypothesis was proposed in 1960s. The Black-Scholes formula (1973) relies on the Brownian random process for price changes. Although the distribution of the price changes can be well approximated near the peak by the Gaussian, it can not lead to the fat tail and this is reflected on the appearance of "smile" which should not be there in reality. ARCH-type models which are ad hoc time-series models and reproduce the fat tails can not satisfy the findings either. They do not have asymptotic power-law decay in the volatility correlation function. The models were introduced to explain clustered volatility where the volatility varies with time.

In recent years a novel effort to solve the problems has appeared, characterized by agents which imitate behaviors of human agents involved in financial markets.[2, 9] The agents-based model[10] seeks to take into account intelligence, learning, complex decision making in addition to emotion, irrationality and intuition. Studies in this direction range from simple models like evolutionary game theory[11] to complicated simulations like the Santa Fe Institute stock market model (SFI model).[12] The SFI model has shown that many of the dynamical properties of real markets such as fat tails and clustered volatility are reproduced automatically when the model allows the participants of the markets to make decisions dynamically. Although successful, it also brought many unanswered questions due to its complexity of simulations.

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6) R. F. Engle, "Fourier Analysis of Distributions Functions. A Mathematical Study of the Laplace-Gaussian Law", Acta Math. 77(1945)1.

7) F. Black and M.Scholes, "The Pricing of Options and Corporate Liabilities", J. Polit. Econ. 81(1973)637.

8) R. C. Merton, "Theory of Rational Option Pricing", Bell J. Econ. Management Sci. 4(1973)141.

9) J. Doyne Farmer, "Physicists attempt to scale the ivory towers of finance", Computing Sci. & Eng., November-December 1999, 26-39.

10)B. LeBaron, "Agent-Based Computational Finance: Suggested Readings and Early Research", J. Economic Dynamics and Control 24(2000)679.

11)J. W. Weibull, "Evolutionary Game Theory", MIT Press, Cambridge, Mass., 1996.

12)B. LeBaron, W. B. Arthur and R. Palmer, "Time Series Properties of an Artificial Stock Market", J. Economic Dynamics and Control 23(1999)1487.

The minority game (MG)[13] offers another approach in terms of simulations. The model is simple, but displays rich behaviors. It is a simplified version of the agents-model, the El Farol's bar attendance problem (BP).[14]

The question of the El Farol problem is whether one should go to the bar of limited seats of 100. If the bar is overcrowded with guests more than 60, one can not enjoy attending the bar. To be able to go there one has to guess when other people do not, based on the past attendance as a public information. The situation is a typical case of people sorting after the limited resource. All what one knows is the past attendance as history. The process of decision making looks similar to heterogeneous agents trying to make the best decision with bounded rationality.

In MG a fixed (odd) number N of agents are given two alternatives, sell or buy for the case of market, route A or route B when deciding over not too crowded route, etc. Which alternatives agents choose, the winners are those who make the same decisions as minority, hence the name minority game. Only information known to the agents is the last M bits of winner's outcome when "0" and "1" is assigned for the alternatives. The length M indicates pre-assigned memory length, called history. The agents have certain number S of strategies to cope with the competition. Every time step each strategy is given a virtual point when it predicts the minority group correctly. Agents use the strategies inductively to play with the best strategy out of S. It involves only three parameters, the number of agents N, the length of memory M and the number of strategies S, yet showing rich structures. The model has been extensively studied numerically as well as analytically.

The main results are summarized as follows.[1), 13)] It shows a self-organized collective behavior signaled by the phase transition between a symmetric phase and an asymmetric phase when $NS \sim N_0 (=2^{M+1})$. The phase transition can be understood in terms of herd effect. Note that the number of possible histories is 2^M and the number of strategies is $2^{\wedge}2^M$. Each strategy can be placed on a Boolean hypercube in the 2^M dimensional strategy space. The similarity between any pair of strategies can be measured by the Hamming distance d_M which counts the number of different digits in each order between two binary digits of

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13)D. Challet and Y.-C. Zhang, "Emergence of Cooperation and Organization in an Evolutionary Game", Physica A **246**(1997)407, Y.-C. Zhang, "Modeling Market Mechanism with Evolutionary Games", Europhys. News **29**(1998)51, Robert Savit, Radu Manuca and Rick Riolo, "Adaptive Competition, Market Efficiency, Phase Transition", Phys. Rev. Lett. **82**(1999)2203, M. A. R. de Cara O. Pla and F. Guinea, "Competition, efficiency and collective behavior in the 'El Farol' bar problem" Eur. Phys. J. B 10(1999)187.

14)Brian W. Arthur, "Inductive Reasoning and Bounded Rationality (The El Farol Problem)", Am. Econ. Assoc. Papers and Proc **84**(1994)406.

same length. The maximal Hamming distance is 2^M which is visualized as completely opposite points on the hypercube, so called anti-correlated pair with the number of strategies N_0. One agent of the anti-correlated pair predicts always the opposite of the other. All the other Hamming distances for this N_0 are 2^{M-1} and they are mutually independent because the Hamming distance is half of the maximal distance. Vast majority of strategies which do not belong to N_0 are positively correlated. We can predict that there should be three distinctive situations according to the relative magnitude of NS and N_0. If NS>> N_0 (crowded case), the players have to use strategies which are positively correlated, necessitating the herd effect (crowd effect) to weaken the adaptability of the players. Resources would be wasted in this case. If, on the other hand, NS<< N_0, the players can choose independent strategies, random strategies. Finally, for the intermediate case NS~N_0, the agents can exploit anti-correlated strategies to coordinate mutual avoidance although they use mostly independent strategies. This is the most efficient case.

The model has been extensively studied in various methods.[1)] Initial results have been obtained by simulations.[13)] Later, it was exactly solved in analogy to the spin-glass in a stationary state.[15)] The dynamical aspects of MG have been clarified by the functional method.[16)]

The model has been extended in several directions.[1)] The crowd-anticrowd theory[17)] was proposed based on a reduced set of strategies related to N_0. It is useful to treat MG intuitively. The model was enlarged to describe the real markets[18)] involving capital, producers, speculators, noise traders, etc. Thermal effects have been considered.[19)] To improve the memory update a neural network was included.[20)] Mixed population of various

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15)D. Challet, M. Marsili and R. Zecchina, "Statistical Mechanics of Heterogeneous Agents", Phys. Rev. Lett. **84**(2000)1824.

16)J. A. F. Heimel and A. C. C. Coolen, "Generating Functional Analysis of the Dynamics of the Batch Minority Game with Random External Information", Phys. Rev. E **65**(2002)016126.

17)Neil F. Johnson, Michael Hart and Pak Ming Hui, "Crowds Effects and Volatility in Markets with Competing Agents", Physica A **269**(1999)1, M. Hart, P. Jefferies, N. F. Johnson and P. M. Hui, "Crowd-Anticrowd model of the Minority Game"(2000), preprint cond-mat/0003486.

18)F. Slanina and Y.-C. Zhang, "Capital Flow in a Two Component Dynamical System", Physica A 272(1999)257, D. Challet, M. Marsili and Y.-C. Zhang, "Modeling Market Mechanism with Minority Game", Physica A **276**(2000)284, Neil F. Johnson, Michael Hart, Pak Ming Hui and Dafang Zheng, "Trader Dynamics in a Model Market", Work presented at the EPS Dublin finance conference (July 1999).

19)A. Cavagna, J.P. Garrahan, I. Giardina and D. Sherrington, "A Thermal Model for Adaptive Competition in a Market", Phys. Rev. Lett. **83**(1999)4429.

20)W. Kinzel, R. Metzler and I. Kanter, "Dynamics of Interacting Neural Networks", J. Phys. A 33(2000)L141, Joseph Wakeling and Per Bak, "Intelligent systems in the context of surrounding environment", Phys. Rev. E **64**(2001)051920.

memories have been treated.[21] It was pointed out that the anti-persistence offers an advantage for agents with longer memory, based on anti-persistent time series on a De Bruijn Graph.[22]

Finally, I discuss the MG-like model recently proposed where an evolutionary effect on strategies is taken into account.[23, 24] The strategies of agents are probabilities p_i, the gene values to follow a predicted outcome after some history. The agent i contradicts the outcome with the probability $1-p_i$. The history of length M and predicted outcome, 0 or 1 are known to all agents as a public information. Similar to the standard MG the minority wins. Each agent gets plus one point (prize) for a winning turn and minus one point (fine) for a losing turn. The probability p_i is updated every time the scores go below a critical value d.

In the present paper I chose a prize point g=0.971 and the fine point fixed as minus one, hence g being equal to the prize-to-fine ratio,[24] and found two other types of gene distribution P(p) for varying d, two peaks for d=-10 and one valley for d=-100 in addition to one peak recently discovered for d=-4.[24] In the simulations 10001 agents have been assumed for the average gene value <p> and the gene distribution P(p), and 100001 agents for PDF f(z) as a function of the change of room occupancy reduced by the standard deviation, z. In all cases 100000 time steps have been taken. The graph for the average gene value indicates that the state is in equilibrium at d=-100 while oscillating at d=-4. These results imply a phase transition as a function of d. We can interpret this as a revival phenomenon to the self-segregation[23] which was induced by decreasing d to overcome an unstable situation. If the tolerance level is sufficiently low, we can take a risk in order to win by the extreme behavior recognized earlier.[23] The room occupancy change shows much wider fluctuation for d=-4 than for d=-100, which is confirmed by PDF. Both PDFs show Gaussian in the central regions, and decay slightly slower than the Gaussian for d=-4 while for d=-100 the curve appears to be taken over by another Gaussian. Although the model exhibits the self-organizing tendency, it does not show the power behavior clearly.

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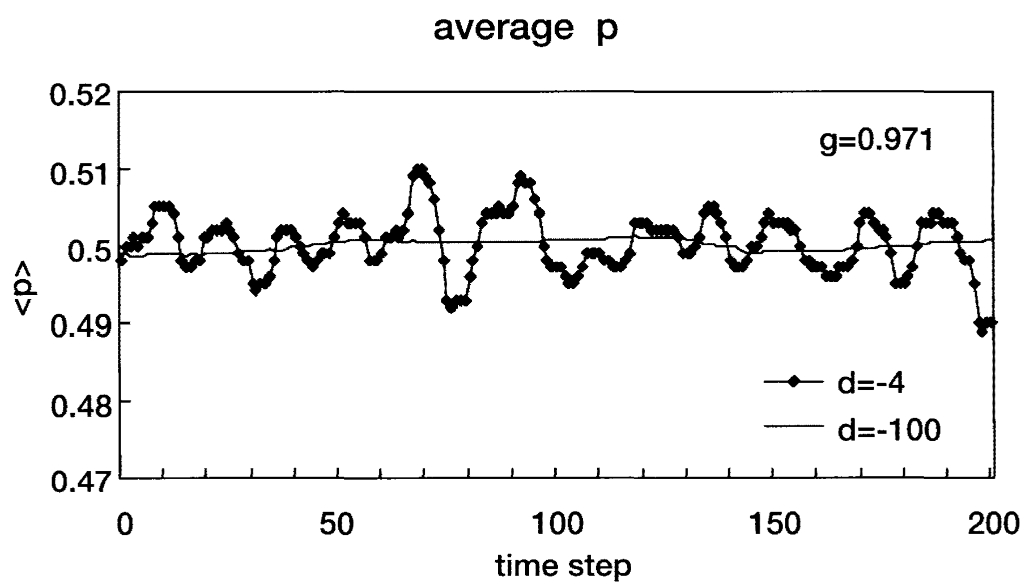
21)N.F. Johnson, P.M. Hui, D. Zheng and M. Hart, “Enhanced winnings in a mixed-ability population playing a minority game”, J. Physics A: Mathematics and General **32**(1999)L427, M. Paczuski and K. E. Bassler, "Self-organized Networks of Competing Boolean Agents"(1999), preprint cond-mat/9905082.

22)R. Metzler, “Antipersistent Binary Time Series”(2001), preprint cond-mat/0109243.

23) Neil F. Johnson, Pak Ming Hui, Rob Jonson and Ting Shek Lo, “Self Organized Segregation within an Evolving Population”, Phys. Rev. Lett. **82**(1999)3360, T. S. Lo, P. M. Hui and N. F. Johnson, “Theory of Evolutionary Minority Game”, Phys. Rev. E **62**(2000)4393.

24)S. Hod and E. Nakar, “Segregation vs. Clustering in the Evolutionary Minority Game”(2002), preprint cond-mat/0201002.

Appendix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d=-4
d=-100
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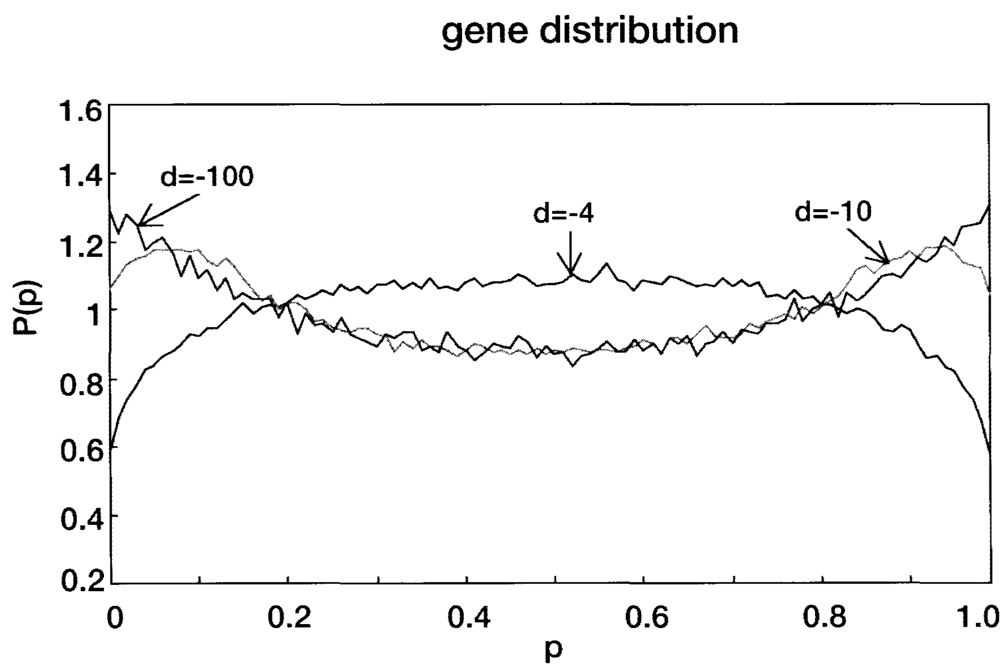
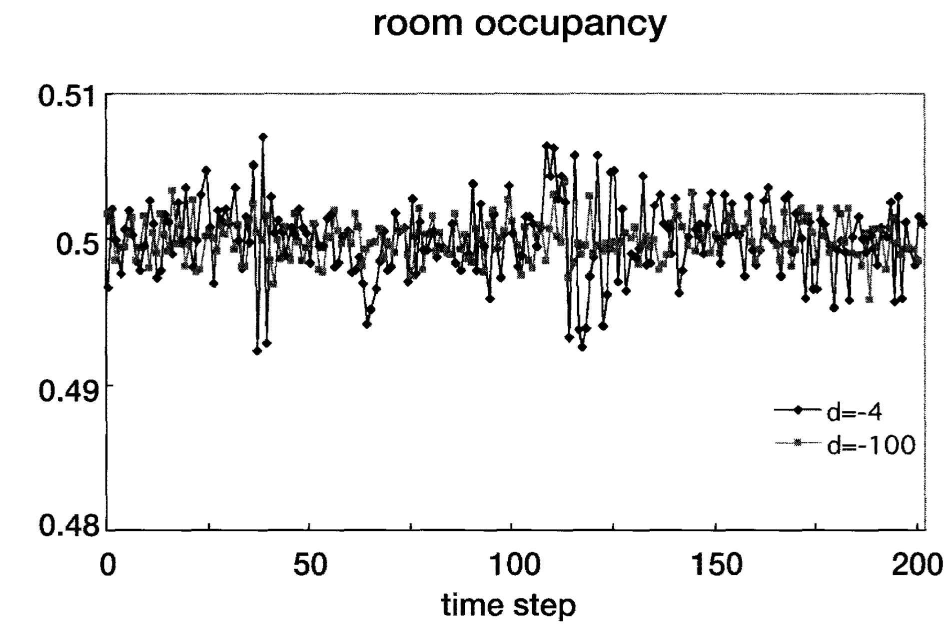
0.51
0.5
0.49
0.48
0.47
0
50
100
150
200
time step


gene distribution
d=-100
d=-4
d=-10
P(p)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
p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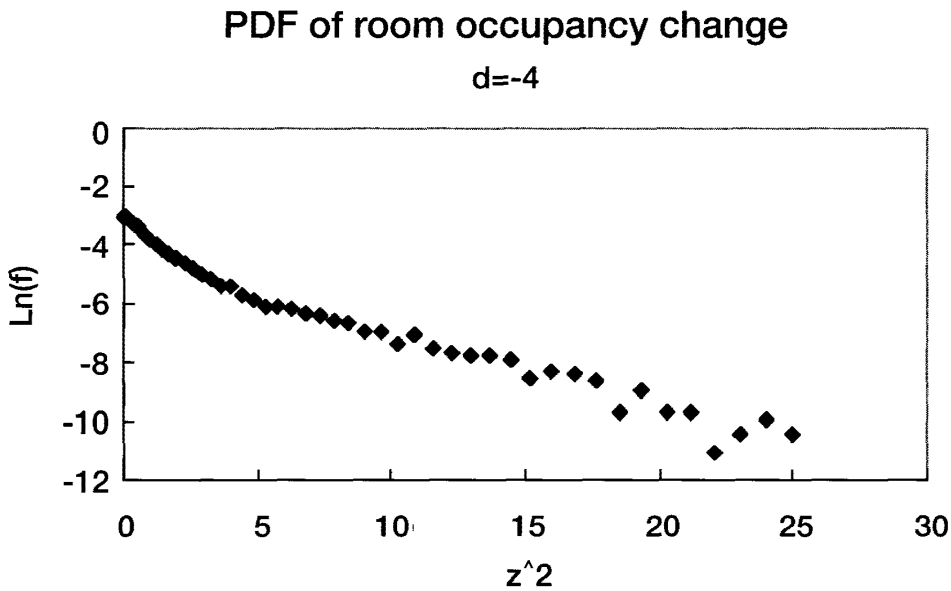
0.51
0.5
0.49
0.48
0
50
100
150
200
time step
d=-4
d=-100


PDF of room occupancy change
d=-4
Ln(f)
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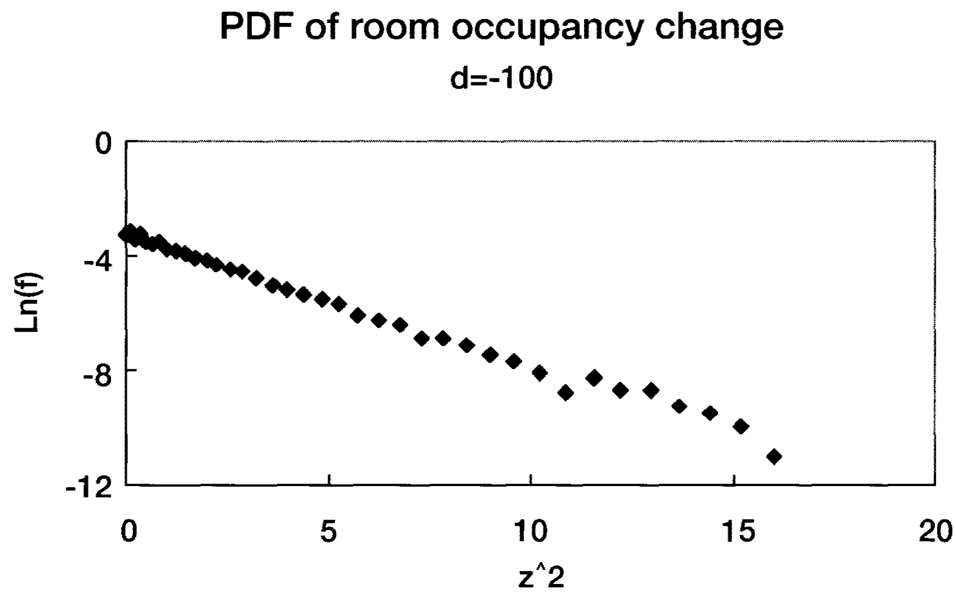
-4
-6
-8
-10
-12
0
5
10
15
20
25
30
z^2


PDF of room occupancy change
d=-100
Ln(f)
0
-4
-8
-12
0
5
10
15
20
z^2